# SELECTION OF AN OPTIMUM MOMENT REDISTRIBUTION IN SEISMIC-RESISTANT DESIGN OF R/C DUCTILE MOMENT RESISTING FRAMES

S.W. Zagajeski and V.V. Bertero


## SYNOPSIS

The results of a study oriented toward the selection of an optimum moment redistribution in seismic-resistant reinforced concrete frame structures are presented. The paper describes a computer-aided optimum inelastic design procedure developed by the authors for the design of reinforced concrete structures which are expected to experience a severe earthquake ground motion during their service lives. By modifying design constraints the proposed design procedure can be used to affect different inelastic moment redistributions.

Three different inelastic designs of a ten-story three-bay frame are presented. In addition, a design based on the results of elastic analysis and a design based on negative moment redistribution allowed by ACI are discussed. In all designs, seismic design forces are found from an inelastic response spectrum.

A comparison of the five designs indicate that moment redistribution has only a minor effect on required material volume. However, redistribution can have a major effect on inelastic rotation demands in response to earthquake ground motions and can relieve steel congestion at beam-column joints by reducing negative design capacities.

The proposed design procedure is shown to be a versatile tool with which to effect inelastic design. Various moment redistributions may be considered by imposing appropriate design constraints while at the same time satisfying serviceability criteria.

## RESUME

Les résultats d'une étude orientée vers la sélection d'une redistribution optimale des moments dans les portiques en béton armé résistant au séisme sont présentés. Cette communication décrit une technique de calcul inélastique optimal par ordinateur développée par les auteurs pour calculer les structures en béton armé qui peuvent être sujettes à de sévères séismes durant leur existence. En modifiant les contraintes de calcul, cette technique peut être employée à calculer les différentes redistributions des moments inélastiques.

Trois calculs inélastiques différents d'un portique de dix étages à trois travées sont présentés. De plus, on discute d'un calcul basé sur les résultats d'une analyse élastique et d'un calcul basé sur la redistribution des moments négatifs permise par l'ACI. Dans tous les cas, les forces de séisme sont calculées à partir d'un spectre de comportement inélastique.

Une comparaison des cinq méthodes indique que la redistribution des moments a seulement un effet mineur sur la quantité de matériaux requis. Cependant, elle peut avoir un effet important sur la capacité de rotation inélastique demandée par les mouvements du sol durant le séisme et peut éliminer l'encombrement d'acier dans les assemblages poutre-colonne en réduisant les moments négatifs.

La méthode proposée semble être une technique flexible capable d'effectuer le calcul inélastique. Différentes redistributions des moments peuvent être considérées en imposant les contraintes appropriées tout en satisfaisant les critères de disponibilité technique.

S.W. Zagajeski received a Ph.D. from the University of California at Berkeley in 1978. He has coauthored several publications dealing with the inelastic behavior and design of reinforced concrete structures. He recently joined the structures group at Colorado State University in Fort Collins, Colorado.

V.V. Bertero received the degrees of both Master of Science (1955) and Doctor of Science (1957) from the Massachusetts Institute of Technology. Currently, he is Professor of Civil Engineering at the University of California, Berkeley. He is the author or co-author of several papers and reports in the area of Earthquake Engineering.

## INTRODUCTION

The accepted general philosophy of seismic-resistant design of buildings other than essential facilities conforms to the principle of comprehensive design (1) in that different design limit states are considered for earthquake ground motions of different severity and frequency of occurrence. Although such a design philosophy is generally accepted as a rational approach to seismic-resistant design, current design methodologies fall short of realizing the objectives of comprehensive design.

For example, consider the design of a building to be constructed near an active fault where a very severe earthquake ground motion is likely to occur during the structure's service life. Although conceptually it is recognized that design is controlled by the ultimate limit states (damageability or collapse), current design practice is typically based on minimum seismic design forces specified by building codes (2) and the results of linear elastic analysis.

The authors believe that structural design should be based on the limit state that controls it. For the above example, since the ultimate limit states are critical, design should be based on an inelastic design procedure which includes moment redistribution in selection of member design forces and takes advantage of the structure's capacity to dissipate energy through large but controlable inelastic deformations.

Some beam moment redistribution is possible following current code stipulations for design of reinforced concrete (R/C) frame structures. Depending on the percentage of member longitudinal reinforcement, the ACI 1977 code (3) allows moment redistribution to a maximum of 20%. The relationship between moment redistribution and steel content is incorporated to ensure that member critical regions (plastic hinge zones) possess sufficient inelastic rotation capacity to attain the assumed redistributions. The ACI limitation on redistribution is believed to be conservative in the case of seismic-resistant R/C frames, however. Design in accordance with present seismic code requirements results in plastic hinge zones characterized by relatively low steel percentages, by the presence of significant compression reinforcement and by close spacing of transverse reinforcement. As a result, such structures should possess sufficient ductility to attain most practical moment redistributions.*

## OBJECTIVE AND SCOPE

The principal objective of this paper is to present and discuss the results of a study oriented toward the selection of an optimum moment redistribution as far as the performance of R/C ductile moment resisting frames (DMRF) during severe earthquake ground motion is concerned. In the investigation various moment redistributions were obtained employing a computer-aided optimum inelastic design procedure developed by the authors for seismic-resistant design of R/C frame structures (5). The design procedure is an extension of a procedure developed previously by Bertero and Kamil (6) for steel DMRF.

The essential features of the design procedure proposed in reference (5) are reviewed first. Three inelastic designs of a ten-story three-bay frame obtained employing the procedure are then presented. In these designs different moment redistributions were affected by modifying the design constraints.

Two additional designs are also presented. In the first design (Design I) moment redistribution is not considered, i.e. member design is based on elastic analysis. In the second (Design II) redistribution of negative moments allowed by ACI is considered.

The five designs are compared with respect to required steel and concrete volumes and with respect to inelastic dynamic response to representative earthquake ground motions.

## INELASTIC SEISMIC-RESISTANT DESIGN PROCEDURE

The authors have previously proposed a computer-aided iterative design procedure for seismic-resistant multistory frame structures (5). The procedure was developed specifically for DMRF constructed of R/C which are expected to experience a severe earthquake ground motion during their service lives. In the following paragraphs the essential characteristics of the procedure are reviewed. Special emphasis is placed on the optimum design problem within the procedure since the formulation of the objective function and the solution of the resulting optimization problem, which becomes nonlinear, have recently been modified. In addition the different moment redistributions discussed in the paper were obtained by modifying the constraints of the optimization problem. Throughout the discussion, reference is made to the design of a ten-story three-bay frame (Fig. 1).

The design procedure consists of five basic steps which are divided into preliminary and final design phases (Fig. 2). In both phases, an

*An upper bound on moment redistribution may be necessary to prevent redistributions which require inelastic rotations greater than typical member rotation capacities. Paulay has suggested an upper bound of 30% (4).

optimum inelastic design which minimizes the volume of flexural reinforcement is found for each story. A weak grider-strong column design philosophy is followed in both design phases in order to limit column inelastic deformations and to prevent formation of soft stories (partial sway mechanisms). In addition, transitions in strength, stiffness and mass through the height, as well as the plan area of the structure, are made as smooth as possible in order to prevent large concentrated inelastic deformations. The discussion to follow will concentrate on the preliminary design phase.

## Preliminary Design Phase

Regardless of how sophisticated the analysis techniques employed in determining member design capacities, the final design will be only as good as the preliminary design used to define seismic forces and to carry out the analysis. In view of this fact the objective of the preliminary design phase is to obtain a preliminary design which is as close as possible to the desired final design. The preliminary phase entails three steps; preliminary analysis, preliminary design, and analysis of the preliminary design. To achieve the stated objective these steps are repeated until the preliminary design is deemed acceptable with respect to established design criteria which reflect the desired characteristics of the final design, and with respect to the dynamic characteristics and ductility factors assumed in evaluating seismic design story shears.

Preliminary analysis--The objective of the first step of the design procedure is to establish design loads and design criteria. On the basis of structure geometry and building function, gravity and wind loads are determined and story masses estimated. Design earthquakes, which are represented by smooth ground motion spectra, are established on the basis of the seismic characteristics of the building site. Ground motion spectra are defined by selected values of effective peak ground acceleration, velocity and displacement (Fig. 3).

Although in previous design examples (5) and in the design examples presented later in this paper only one design earthquake (corresponding to a severe gound motion) has been considered, additional ground motion spectra corresponding to different design limit states (for example the serviceability limit state) can be used.

Based on assumed (or computed if a preliminary design has already been obtained) structure dynamic characteristics, design story shears are obtained from an inelastic response spectrum (Fig. 3) employing a modal analysis technique. In the designs presented the inelastic spectrum is constructed from the ground motion spectrum following a procedure suggested by Newmark (7). Other response spectra can be used when they become available.

The story shears obtained by the modal analysis technique are finally modified to account for the P-$\Delta$ effect. Although the use of a modal analysis technique for an inelastic multi-degree of freedom system is in general not correct, it is considered to be a significant improvement over current code seismic force specifications (8).

Preliminary design--Once the gravity and seismic design forces have been established, the beam and column sizes and flexural reinforcement are found as part of the preliminary design step. Member design is based on the solution of an optimization problem formulated for each story employing the design subassemblage in Fig. 4. Imposing a weak grider-strong column design criterion reduces the problem to one of finding beam moment capacities. The optimum design problem may be summarized as follows:

Find the beam design moments

$$M_i > 0 \qquad i = 1,N \tag{1a}$$

which minimize the objective function

$$C(M_i) > 0 \tag{1b}$$

and which satisfy the design constraints

$$G_j(M_i) \geq 0 \qquad j = 1,NC \tag{1c}$$

where

N = the number of desired design moment capacities and

NC = the number of design constraints.

Three sets of design constraints are imposed. Equilibrium constraints, imposed to ensure a safe design, are derived from the kinematic theorem of simple plastic theory. Serviceability constraints, imposed to protect against yielding and excessive cracking and deformation under service load conditions, are based on the results of elastic analysis for service level loads. Finally a series of practical constraints are imposed to meet code requirements and to obtain a practical design.

Various moment redistributions may be effected by modifying the practical design constraints. In this study three different inelastic designs have been obtained in this way. In Design III the positive moment capacity at a given support section is constrained to be at least one-half the corresponding negative capacity. This requirement is stipulated by ACI (3). In Design IV the positive and negative moment capacities at a given support section are constrained to be equal. In Design V the design moment capacities on either side of an interior column are constrained to be equal. For the subassemblage in Fig. 4 the following constraints result

$$\begin{aligned} M_3^- &= M_4^- \\ M_3^+ &= M_4^+ \end{aligned} \tag{2}$$

The constraints defined by equation 2 eliminate bar curtailment at interior beam-column joints and thus relieve steel congestion in these joints.

It should be noted that all design constraints are linear.

The objective function, which is proportional to the volume of flexural reinforcement, is based on the following approximate relationship between the beam design capacity at section i, $M_i$, and the corresponding steel area $A_{s_i}$

$$M_i = A_{s_i} f_y \, jd \tag{3}$$

where

$f_y$ = nominal steel yield stress

$jd$ = lever arm between the resultant internal tensile and compressive forces.

Separating the contributions of the beam and column reinforcements the objective function may be written as

$$C(M_i) = C_C(M_i) + C_B\,(M_i) \tag{4}$$

where

$C_C\,(M_i)$ accounts for the column reinforcement

$C_B\,(M_i)$ accounts for the beam reinforcement

The column term $C_C\,(M_i)$ is determined on the basis of the weak girder-strong column design criterion. The sum of the column moments at a beam-column joint are expressed in terms of the desired beam design capacities by considering joint equilibrium. As shown in reference 5, $C_C\,(M_i)$ is a linear function of $M_i$.

The beam term $C_B\,(M_i)$ is based on equation 3. Since $M_i$ is linearly related to $A_{s_i}$ the volume of flexural reinforcement in a given span is proportional to the area under the design moment envelope (Fig. 5). For the $k^{th}$ span

$$[C_B(M_i)]_k = -\int_0^{B_1} \mathcal{M}_1\,(x,M_i)dx - \int_{B_2}^{L} \mathcal{M}_2\,(x,M_i)dx + \int_{B_3}^{L} \mathcal{M}_1(x,M_i)dx + \int_0^{B_4} \mathcal{M}_2\,(x,M_i)dx + \int_{B_3}^{B_4} \mathcal{M}_3\,dx + xM\cdot(B_2-B_1) + F_A\,(M_i) \tag{5}$$

where

$$M_i > 0 \qquad i = 1,5$$

$$\mathcal{M}_1 = -M_1 + \frac{WLx}{2} + \frac{(M_1 + M_5)x}{L} - \frac{Wx^2}{2}$$

$$\mathcal{M}_2 = M_2 + \frac{WLx}{2} + \frac{(M_2 + M_4)x}{L} - \frac{Wx^2}{2}$$

$$\mathcal{M}_3 = M_3$$

$$W = 1.2\ D.L + 1.0\ L.L.$$

$$XM = 1/4\ MAX\ (M_1, M_4) \quad *$$

$$B_1 = x\ @\ \mathcal{M}_1(x, M_i) = XM$$

$$B_2 = x\ @\ \mathcal{M}_2(x, M_i) = XM$$

$$B_3 = x\ @\ \mathcal{M}_1(x, M_i) = M_3$$

$$B_4 = x\ @\ \mathcal{M}_2(x, M_i) = M_3$$

$F_A(M_i)$ = a linear function in $M_i$ which depends on the development length and column width.

The term $C_B\ (M_i)$ for the entire design subassemblage is found by summing $[C_B\ (M_i)]_k$ for each span.

It should be noted that the negative sign in the first two terms is required because the indicated integration yields a negative area.

On examination of equation 5 it is evident that $C_B\ (M_i)$ is nonlinear. Consequently, a nonlinear solution technique is required to solve the optimum design problem. Before discussing the technique used, some comments on the form of $C_B\ (M_i)$ are made.

The authors believe that an accurate measure of the volume of beam flexural reinforcement requires consideration of realistic steel detailing, such as bar development lengths and bar curtailment. It is felt that the function defined by equation (5) provides an adequate representation of these details. In addition the function is differentiable, an essential characteristic of functions defining a nonlinear optimization problem since nonlinear solution techniques depend on gradients of these functions.

The nonlinear solution technique employed to solve the optimization problem described above is the cutting plane method (9). This technique applies linear programming through a sequence of local linearizations to obtain the minimium of a convex function of real variables subject to convex constraints. In the method it is assumed that the constraints confine the solution to a bounded set. The discussion which follows is limited to linear constraints.

In the cutting plane method the optimization problem defined by equation (1) is replaced by the equivalent problem of minimizing a new variable, Z subject to constraints

$$Z \geq C\ (M_i) \qquad (6)$$

*Based on ACI A.5.5 which requires at least one-fourth of the negative reinforcement be continuous throughout the top of the member.

$$g_{ij}\ M_i \geq b_j \tag{7}$$

where

$g_{ij}$ = coefficient of the $i^{th}$ design variable in the $j^{th}$ constraint (since the constraints are linear $g_{ij}$ is a constant)

$b_j$ = a constant defining the $j^{th}$ design constraint

In the solution of this problem the nonlinear constraint defined by equation (6) is linearized by the Taylor series approximation. Details of the solution algorithm as applied to the optimum design problem presented here may be found in reference 10.

It should be noted that initiation of the cutting plane method requires an initial or starting design vector which satisfies the design constraints. The starting vector used in this study is based on the results of elastic analyses.

The beam design moment capacities found by solving the three inelastic optimum design problems described above are summarized in Table 1. Beam design capacities found on the basis of elastic analysis (Design I) and capacities found from elastic analysis considering code allowed moment redistribution (Design II) are also shown.

In Design II the magnitude of redistribution, MR, was controlled by the ACI expression

$$MR = 20\ \left(1 - \frac{\rho - \rho'}{\rho_b}\right) \tag{8}$$

where

$\rho$ - is the tension reinforcement ratio ($A_s$/bd)

$\rho'$ - is the compression reinforcement ratio ($A'_s$/bd) and

$\rho_b$ - is the reinforcement ratio corresponding to simultaneous steel yielding and concrete failure ($\varepsilon_{max}$ = 0.003)

Moment redistribution defined by equation (8) was applied independently to each span of the design subassemblage in Fig. 4.

A comparison of the beam design capacities presented in Table 1 indicate the following:

1. Negative design capacities in the exterior span from floor level 8 to level 2 in designs III and V are equal to elastic design moments. This is attributed to the form of the objective function which tends to put less weight on negative design capacities than on positive design capacities (Fig. 5). Consequently, negative design moments tend toward their upper bound, the ordinate of the ultimate load elastic moment envelope.

2. Negative design capacities decrease in the interior span for all inelastic designs. The largest reductions occur in Design IV. For example at floor level 2 the negative moment for the interior span decreased from an elastic value of 1248 kN-m to 813 kN-m, a redistribution of approximately 35%. The reduction in negative moment capacities in the interior span is attributed to the fact that the interior span length is longer than that in the exterior span. Consequently the objective function places greater weight on the design capacities associated with the interior span than on those associated with the exterior span.

3. A comparison of Designs I, III and V indicate that the inelastic design procedure results in a significant reduction in positive moment capacities at sections 1 and 4. This is attributed to the character of the objective function which as indicated previously places a larger weight on positive design capacities than on negative capacities. As a result the positive capacities at sections 1 and 4 tend toward their minimum value, one-half the corresponding negative capacity.

One attractive feature of the inelastic optimum design procedure is that it provides the designer with a versatile tool to affect moment redistribution. By specifying appropriate design constraints a designer may obtain almost any desired redistribution. For example if the 35% redistribution found for Design IV is considered too high a new constraint could be added to limit the redistribution to the desired magnitude.

Once the inelastic optimum design problem has been solved for the beam design moment capacities, beam and column sizes and flexural reinforcement are found with the aid of a digital computer. The beam sizes and flexural reinforcement required to resist the optimum beam design moments, modified to account for column slenderness effects and code capacity reduction factors, are found first. Column design forces are then determined on the basis of the weak girder-strong column design criterion and gravity load conditions and the column cross sections and reinforcement are found.

Automated member design is considered an attractive feature of the design procedure. The computer relieves the designer of a tedious and time consuming computational chore, thus freeing him/her to act creatively in the design process. In addition, this computational tool allows the designer to generate, in a relatively short period of time, several alternative designs which can be used as guidelines for the final design.

Preliminary design results are summarixed in Tables 2 and 3. The inelastic design results were obtained after two iterations.

Beam and column sizes are given in Table 2. In determining member sizes the following design constraints were imposed.

(a) Beam and column sizes (except for the first story columns) were constrained to be the same for at least two stories.

(b) In order to achieve a smooth transition in stiffness through the frame height the increment in beam and column depth was set at 40 mm.

(c) Selection of column size was constrained by the criterion that the axial load be less than the balanced failure load.

The last constraint caused column size to be controlled by axial load. Since the design axial loads were essentially the same for all designs one set of column sizes was obtained.

Two different sets of beam sizes resulted. The beam depths in Designs II and IV were 40 mm smaller than those in Designs I, III and V because of a reduction in negative design capacities caused by moment redistribution.

Table 3 summarizes the concrete and steel volumes for the five designs. The steel volumes presented in Table 3 were computed on the basis of equation (5) and provide only a qualitative measure of the required steel.

The variation of steel volume between the various designs is small, the maximum difference being 5% between Designs I and II.

Although moment redistribution has only a minor effect on required steel volume, it can reduce negative design capacities (Designs II and IV) thus decreasing beam sizes and relieving steel congestion at beam-column joints. In addition reduced negative design capacities typically result in larger positive capacities. The resulting increase in compression steel associated with negative design sections (Table 4) will typically increase the inelastic deformation capacity.

It should be noted that formulation of the optimization problem requires the results of elastic analysis. Consequently an initial or starting design is required. Various methods to determine a starting design are presented in reference 5. Typically member sizes found on the basis of the optimization solution are different than those used in the formulation of the design problem. As a result the preliminary design step is repeated until the member sizes before and after optimization are the same.

<u>Analysis of the preliminary design</u>--Once a preliminary design has been obtained, a series of elastic and inelastic analyses are carried out in order to determine the acceptability of the design. Elastic analysis are carried out to determine dynamic characteristics which are compared to those assumed in evaluating seismic design forces. In addition, response under service load conditions is evaluated.

Inelastic static analyses are carried out to determine the structure's strength and to reveal apparent weaknesses in the design which would be indicated by large localized inelastic deformations or significant column yielding.

Finally, a series of nonlinear time history analyses are carried out to evaluate the structure's response to representative earthquake ground motions. Response envelopes are examined to determine whether the indicated inelastic deformations are acceptable with respect to established story drift limitations and with respect to expected member deformation capacity.

On the basis of the data generated by these analyses the designer determines the acceptability of the preliminary design. If the design is considered acceptable, the final design phase is entered, if not, the three steps defining the preliminary design phase are repeated. Important analytical results for the five designs defined above are reviewed in the following paragraphs.

Results of elastic frequency analyses indicate that the effect of the different beam sizes on frame dynamic characteristics was small. For example the first mode period, $T_1$, of Design I was 1.08 sec and that of Design II, 1.13 sec. The analytical model used to determine $T_1$ includes an approximation of the stiffness associated with the floor slab (11).

The nonlinear static behavior of Designs I thru IV is illustrated in Fig. 6. The behavior of Design V is essentially the same as that of Design III.

In the nonlinear static analysis each frame was subjected to design gravity loads and a monotonically increasing seismic base shear which was distributed through the height of the frame according to the lateral force pattern obtained from modal analysis. The effect of $T_1$, on seismic design forces was considered in Designs III-V. Consequently, the design base shear for Design IV is smaller than that for Designs III and V. However, since the beam design capacities in Design II were obtained by applying code allowed moment redistribution to the results of elastic analysis for Design I, the design base shear for Designs I and II were the same.

Significant overstrength is evident in all designs ranging from 45% for Design III to 56% for the elastic design. Two factors contribute to the overstrength. First the final beam strengths were larger than required because of capacity reduction factors and slenderness amplifications factors. Second the frame did not transform into a mechanism simultaneously as assumed in evaluation of design forces. Instead, hinge formation was gradual.

A comparison of behavior indicates that, as the reduction in negative design capacity associated with moment redistribution increases (In Design I there was no reduction and in Design IV the reduction was a maximum), the departure from the elastic loading curve is more gradual. This reflects the earlier yielding of negative moment sections for designs with reduced negative design capacities.

The nonlinear dynamic response to the El Centro (EC) N.S. component and Derived Pacoima Dam (DPD) ground motions is summarized in Figs. 7-10. In both ground motions the peak ground acceleration was 0.4g.

A comparison of story displacement and story drift envelopes (Figs. 7-8) indicates only minor differences in behavior among the various designs. For example, the roof displacement in Designs I and IV differed by less than 10% in response to the Pacoima ground motion. However, although the maximum ground accelerations of the two acceleration records were the same, a significant difference in response is evident. Story displacements and story drifts for the DPD ground motion were approximately three times those for the EC motion. This demonstrates the need to consider all possible ground motions at a given site and also all characteristics of these ground motions (not just peak ground acceleration) when selecting a design earthquake (12).

The effect of moment redistribution on local inelastic behavior is illustrated in Figs. 9 and 10. Accumulated beam plastic rotations, $\theta_p^{ACC}$, defined as the absolute sum of all plastic rotations at a given section, are presented for both exterior and interior spans.

A comparison of $\theta_p^{ACC}$ data for the various designs indicates that the designs obtained employing the proposed inelastic design procedure (Designs III-V) require larger inelastic deformation capacities than the designs based on ACI (3) design provisions (Designs I and II). Design IV in which the reduction in negative design capacities was a maximum experienced the most significant inelastic behavior. For example using Design I as a benchmark the maximum $\theta_p^{ACC}$ for the interior span increased by more than 20% in response to DPD (0.044 to 0.053) and by nearly 40% in response to EC (0.020 to 0.028).

The larger inelastic deformation demands for Designs III, IV and V reflect an essential difference between the code allowed redistribution and the moment redistribution obtained employing the proposed inelastic design procedure. Moment redistribution in Design II was obtained on the basis of elastic moment envelopes which were constructed by considering specified load combinations (5). These load combinations reflect the effect of partial loading.*

The design moment at a given section is the critical value (minimum for negative design capacity, maximum for positive design capacity) found considering all load combinations. Consequently the positive and negative design capacities at a given section do not in general correspond to the same loading condition. For example at a typical support section in a lower story, the negative design capacity is based on the load combination

$$U = 1.2D + 1.0L + 1.0E \tag{9}$$

where

D = dead load

L = live load

E = seismic load

*The effect of pattern or checkerboard loading has not been considered. However, the ratio of live load to dead load is small (0.19) and the effect of pattern loading should be minimal.

while the corresponding positive capacity is based on

$$U = 0.8D - 1.0E \tag{10}$$

This is in contrast to the proposed design procedure in which redistribution is obtained through an optimization technique. Equilibrium is incorporated into the procedure by imposing a series of constraints formulated on the basis of simple plastic theory. For each possible failure mechanism of the selected design subassemblage a constraint of the form

$$\theta_{ij} M_j \geq \omega_i \tag{11}$$

where

$M_j$ = design capacity at section j

$\theta_{ij}$ = virtual plastic hinge rotation at section j in the $i^{th}$ failure mechanism

$\omega_i$ = virtual work done by external forces in the $i^{th}$ mechanism

is imposed. The external work term is based on full factored loads since this represents the most critical case, and the effect of partial loading on member design forces is accounted for by moment redistribution.

The effect of these different design models on moment redistribution is illustrated by the design moment capacities presented in Table 1. If the sum of design capacities at a given story is used as a norm to compare the various designs, it is evident that the sum for Designs III-V is smaller than that for Designs I and II. The smaller values for the optimum inelastic designs reflect the redistribution which accounts for partial loading and explains why these designs experienced larger inelastic deformation demands.

## Final Design Phase

The objective of the final design phase is to arrive at the optimal solution to the seismic design problem. Seismic design forces are determined utilizing characteristics of the structure found in the preliminary design phase. These forces are then used in conjunction with a more sophisticated subassemblage to formulate the optimization problem from which the final design is obtained. Once a design has been obtained, a series of analyses is carried out to check the overall reliability of the design and to provide guidelines for detailing to ensure ductile behavior.

The final design subassemblage is shown in Fig. 11. In this subassemblage, the column mid-height inflection point assumed in the preliminary design (Fig. 4) has been eliminated. In addition, more design parameters are involved than in the preliminary design subassemblage, which should provide a more uniform distribution of moment capacities.

## CONCLUSIONS AND RECOMMENDATIONS

The optimum inelastic design procedure proposed by the authors provides the designer with a versatile tool to effect inelastic design. Various moment redistributions may be considered by imposing appropriate design constraints while at the same time satisfying serviceability criteria.

The design model employed by the optimum inelastic design procedure is more realistic with respect to moment redistribution than that recommended by ACI (1977). Design capacities in the proposed procedure are based on strength criteria at ultimate load conditions. The effects of partial loading are accounted for by moment redistribution. This is in contrast to the ACI 'inelastic' design capacities which are determined from elastic moment envelopes constructed on the basis of load combinations which do not occur at the same time.

It is felt that for seismic resistant R/C ductile frames an optimum moment redistribution should reduce negative design capacities. Although an objective function proportional to the volume of flexural reinforcement typically results in redistributions contrary to this optimum, the results for design IV demonstrate that a reduction in negative design capacities may be effected within the context of the optimum inelastic design procedure.

A comparison of inelastic rotation demands indicates that designs found employing the proposed inelastic design procedure (Designs III-V) typically require larger deformation capacities than designs based on elastic moment envelopes (Designs I and II). The comparatively large deformation demands for Design IV, which experienced the largest reduction in negative design capacities, suggests imposing a practical design constraint which would limit the amount of moment redistribution. The merits of such a constraint, as well as new constraints formulated to optimize moment redistribution with respect to seismic inelastic deformation demands, should be investigated.

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TABLE 1 Beam Design Capacities


Section Identification

| Floor Level | Section | DESIGN (Negative/Positive)* | | | | |
|---|---|---|---|---|---|---|
| | | I | II | III | IV | V** |
| Roof | 1 | 246/123 | 209/105 | 289/289 | 246/246 | 289/289 |
| | 3 | 326/163 | 277/139 | 289/289 | 246/246 | 371/289 |
| | 4 | 538/269 | 456/229 | 371/289 | 370/370 | - |
| 10 | 1 | 519/246 | 418/209 | 519/289 | 462/462 | 519/289 |
| | 3 | 459/229 | 388/213 | 392/289 | 325/325 | 480/289 |
| | 4 | 710/355 | 604/302 | 477/289 | 468/468 | - |
| 9 | 1 | 662/337 | 533/362 | 662/337 | 587/587 | 662/337 |
| | 3 | 613/344 | 506/392 | 528/337 | 426/426 | 636/337 |
| | 4 | 871/435 | 740/370 | 675/337 | 567/567 | - |
| 8 | 1 | 755/389 | 604/462 | 755/377 | 662/662 | 755/377 |
| | 3 | 691/433 | 564/502 | 688/521 | 472/472 | 712/436 |
| | 4 | 938/469 | 792/442 | 723/362 | 604/604 | - |
| 7 | 1 | 851/476 | 678/573 | 851/426 | 745/745 | 851/462 |
| | 3 | 802/524 | 632/616 | 802/581 | 530/530 | 802/401 |
| | 4 | 1041/520 | 872/562 | 800/400 | 666/666 | - |
| 6 | 1 | 900/523 | 720/625 | 900/450 | 780/780 | 900/500 |
| | 3 | 848/570 | 672/670 | 848/632 | 554/554 | 848/564 |
| | 4 | 1064/532 | 889/593 | 929/464 | 676/676 | - |
| 5 | 1 | 996/619 | 806/720 | 996/498 | 865/865 | 996/533 |
| | 3 | 946/662 | 770/770 | 946/700 | 621/621 | 946/616 |
| | 4 | 1158/639 | 962/702 | 1008/504 | 737/737 | - |
| 4 | 1 | 1044/681 | 852/792 | 1044/522 | 904/904 | 1044/624 |
| | 3 | 1013/715 | 820/820 | 1013/892 | 720/720 | 1013/689 |
| | 4 | 1178/639 | 977/729 | 1067/534 | 748/748 | - |
| 3 | 1 | 1164/801 | 962/918 | 1164/623 | 1011/1011 | 1164/672 |
| | 3 | 1143/826 | 942/942 | 1143/902 | 814/814 | 1143/718 |
| | 4 | 1271/723 | 1049/833 | 1149/575 | 809/809 | - |
| 2 | 1 | 1178/902 | 1005/1005 | 1178/589 | 1011/1011 | 1178/715 |
| | 3 | 1263/886 | 1080/986 | 1263/1112 | 956/956 | 1263/804 |
| | 4 | 1291/754 | 1063/872 | 1255/628 | 813/813 | - |

* Units are kN-m

** $M_3 = M_4$

TABLE 2 Member Sizes

| Story | Column Sizes (mm) | | Beam Sizes (mm) | |
|---|---|---|---|---|
| | Exterior | Interior | Designs I, II, V | Designs II, IV |
| Roof | 520 | 680 | 400 x 800 | 380 x 760 |
| 10 | 560 | 720 | 400 x 800 | 380 x 760 |
| 9 | 560 | 720 | 420 x 840 | 400 x 800 |
| 8 | 600 | 760 | 420 x 840 | 400 x 800 |
| 7 | 600 | 760 | 440 x 880 | 420 x 840 |
| 6 | 640 | 800 | 440 x 880 | 420 x 840 |
| 5 | 640 | 800 | 460 x 920 | 440 x 880 |
| 4 | 680 | 840 | 460 x 920 | 440 x 880 |
| 3 | 680 | 840 | 480 x 960 | 460 x 920 |
| 2 | 720 | 880 | 480 x 960 | 460 x 920 |

NOTE: columns are square

TABLE 3 Material Volumes

| Design | Steel Volume ($m^3$) | Concrete Volume ($m^3$) |
|---|---|---|
| I | 2.35 | 153 |
| II | 2.23 | 146 |
| III | 2.26 | 153 |
| IV | 2.31 | 146 |
| V | 2.27 | 153 |

TABLE 4 Beam Reinforcement Floor Level 3*

| Design | Section 1 | | Section 3 | | Section 4 | |
|---|---|---|---|---|---|---|
| | negative | positive | negative | positive | negative | positive |
| I | 10 | 7 | 10 | 7 | 11 | 6 |
| II | 8 | 8 | 8 | 8 | 9 | 7 |
| III | 10 | 5 | 9 | 7 | 9 | 4 |
| IV | 9 | 8 | 7 | 7 | 7 | 7 |
| V | 10 | 5 | 9 | 5 | 9 | 5 |

* number in Table is number of #8 bars

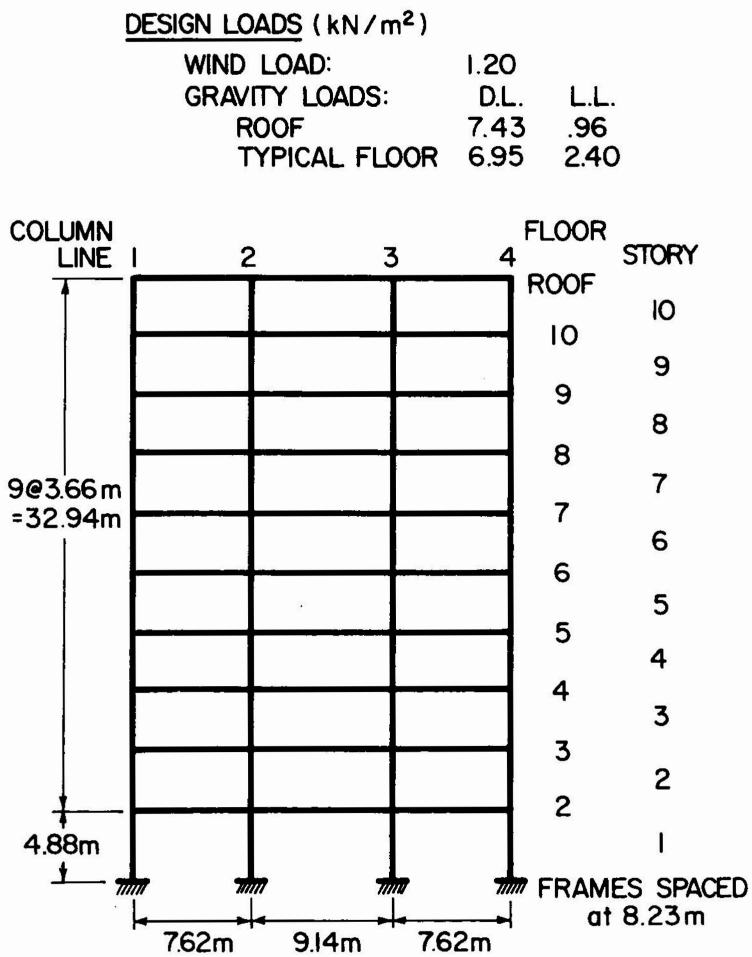


FIG. 1 DESIGN EXAMPLE

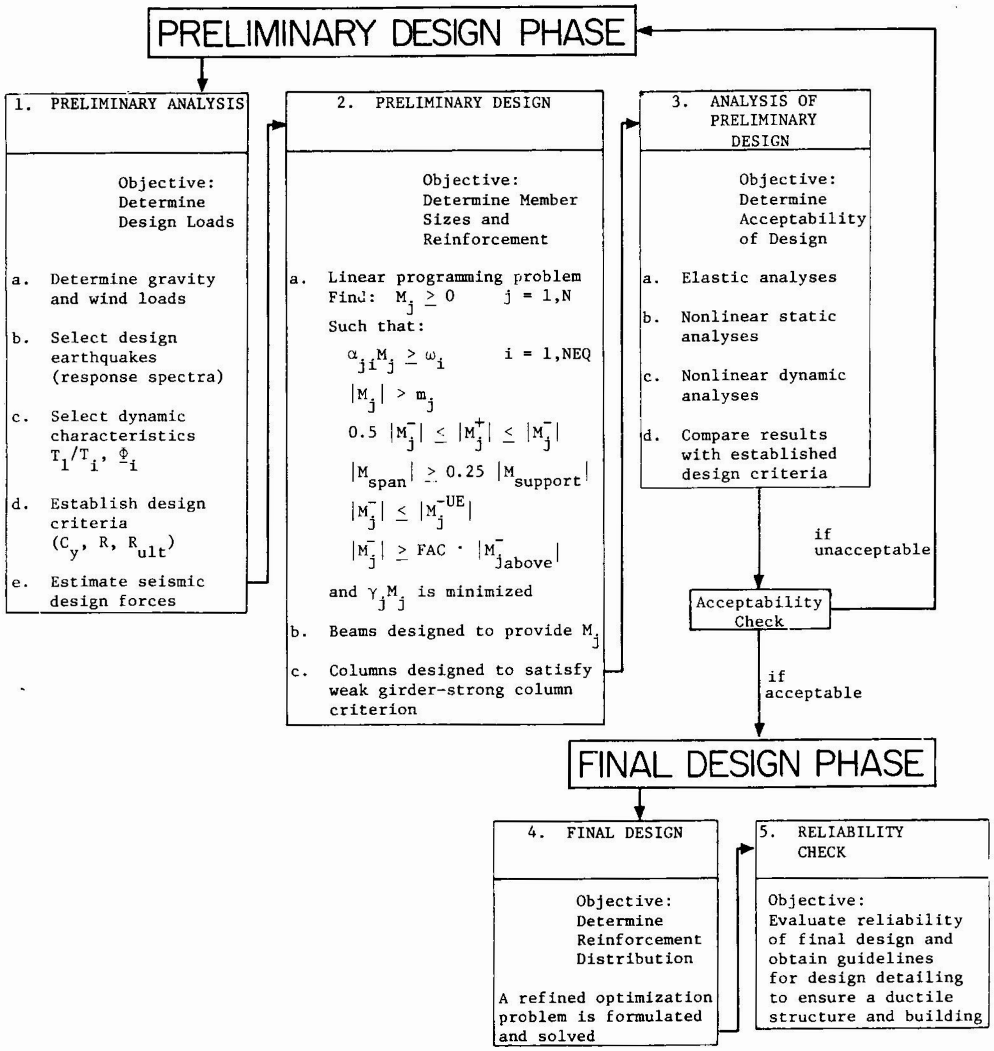


FIG. 2 FLOW CHART OF DESIGN PROCEDURE

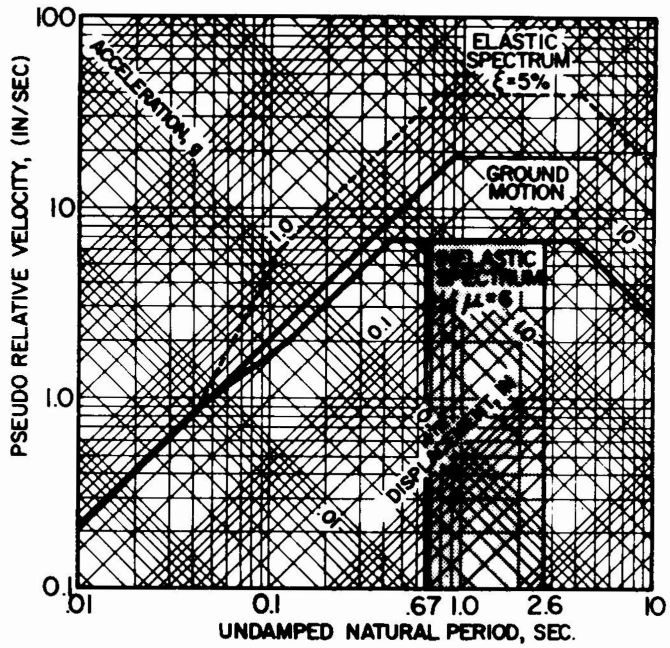


FIG. 3 DESIGN SPECTRA (1 in. = 25.4 mm)

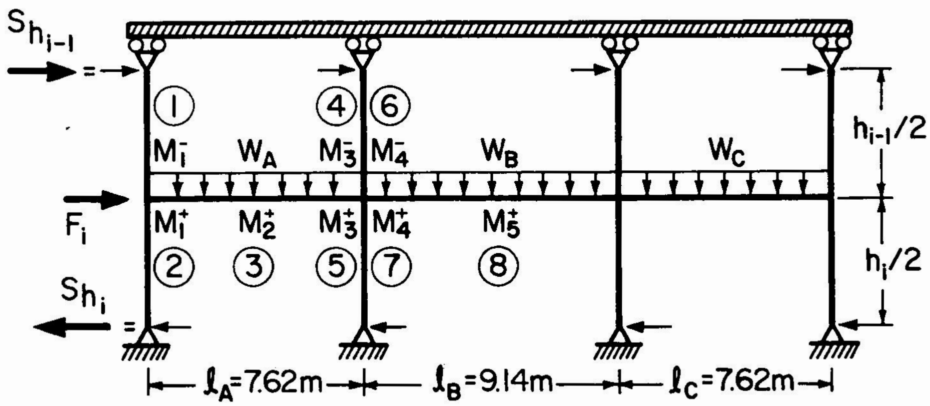


FIG. 4 SUBASSEMBLAGE FOR PRELIMINARY DESIGN

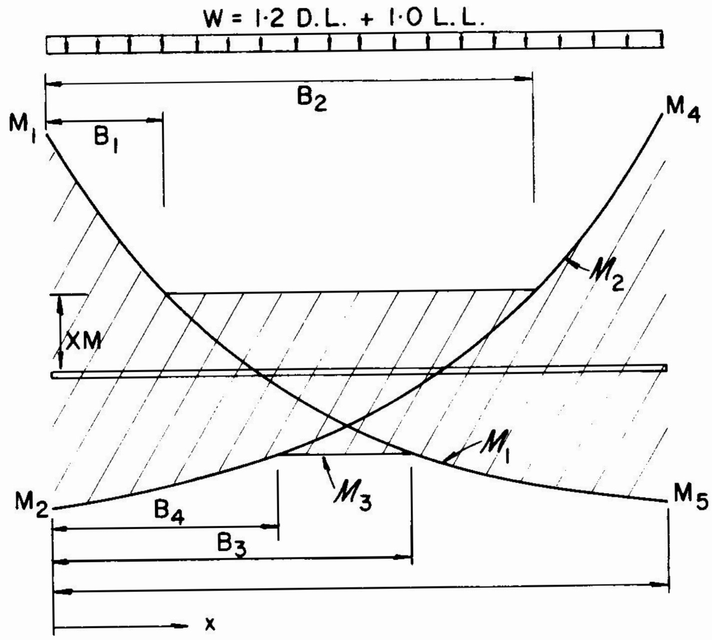


FIG. 5 TYPICAL DESIGN MOMENT ENVELOPE

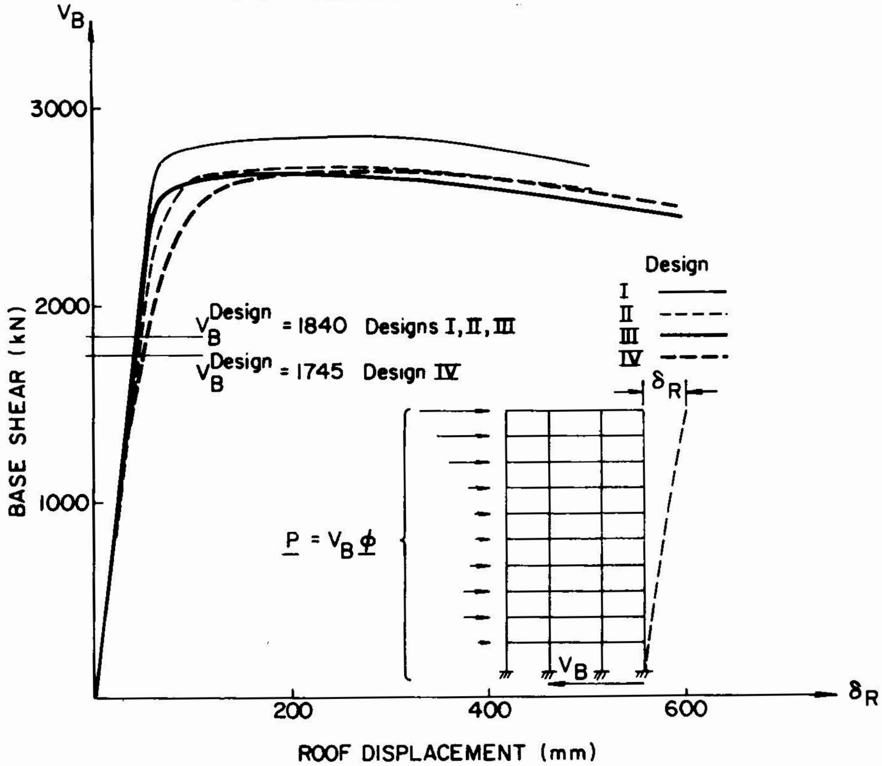


FIG. 6 NONLINEAR STATIC BEHAVIOR

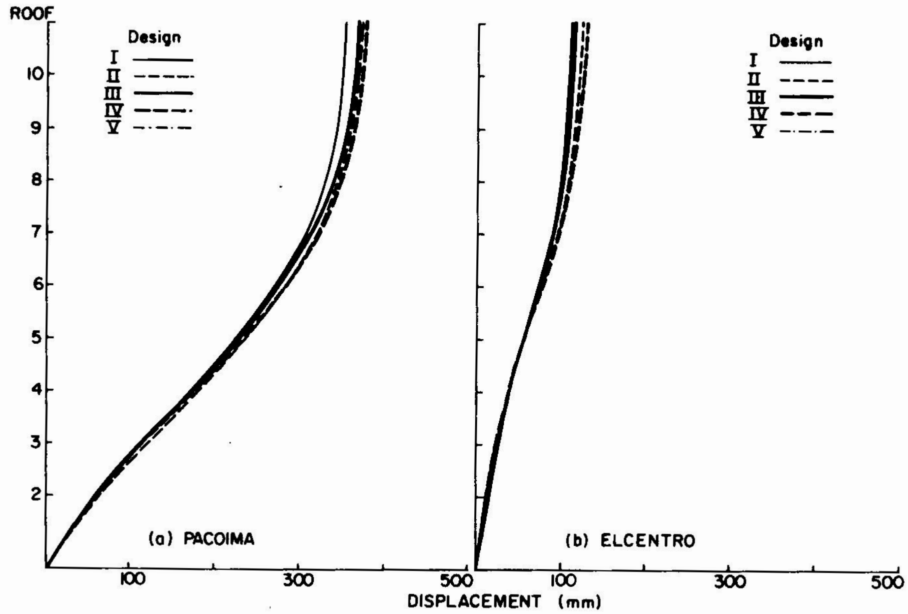


FIG. 7 STORY DISPLACEMENT ENVELOPES

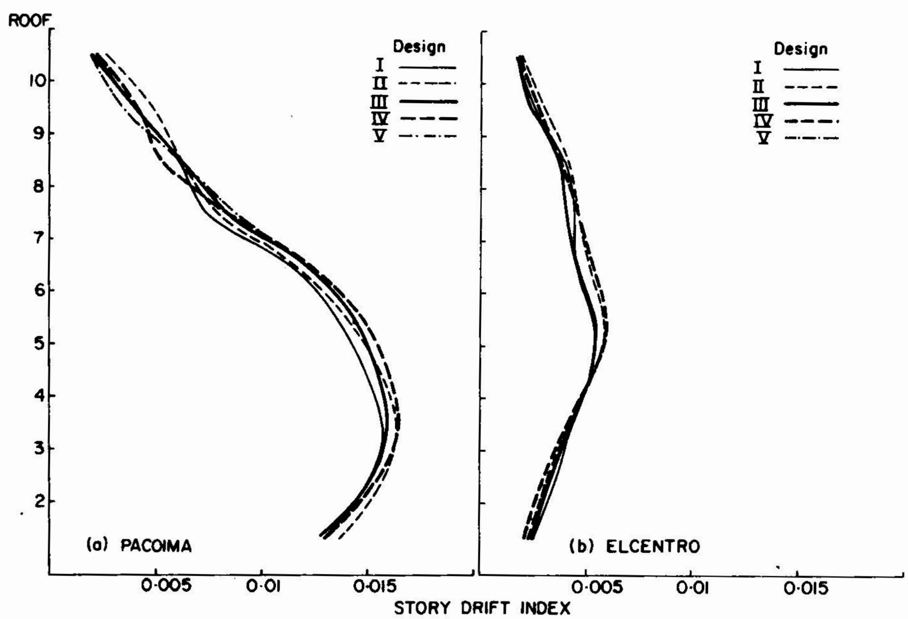


FIG. 8 STORY DRIFT INDEX ENVELOPES

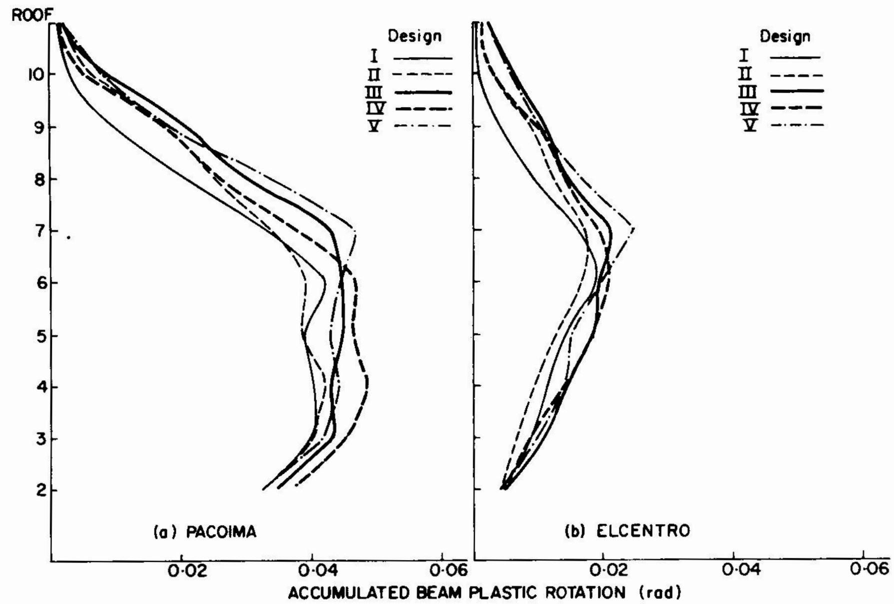


FIG. 9 ACCUMULATED BEAM PLASTIC ROTATIONS, EXTERIOR SPAN

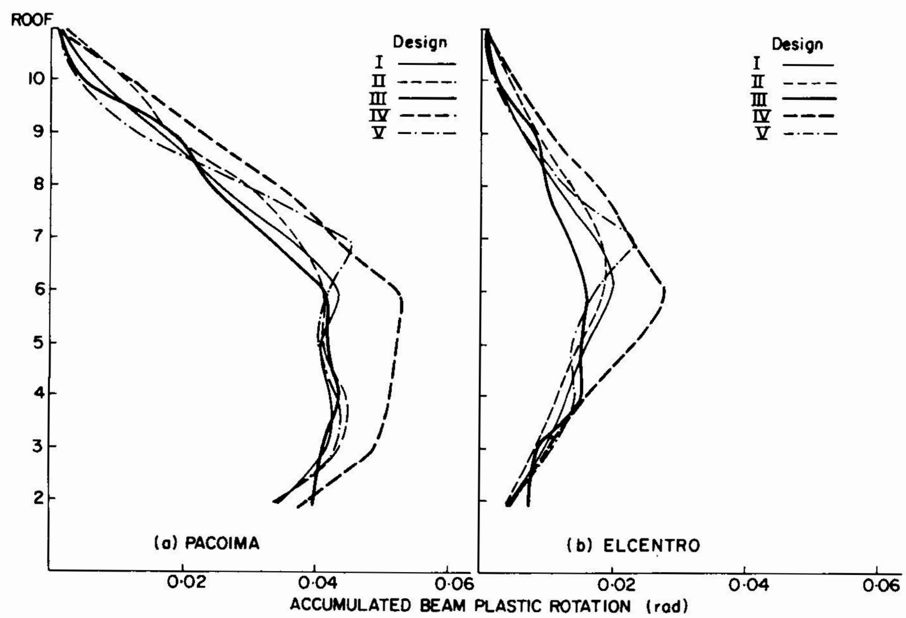


FIG. 10 ACCUMULATED BEAM PLASTIC ROTATIONS, INTERIOR SPAN

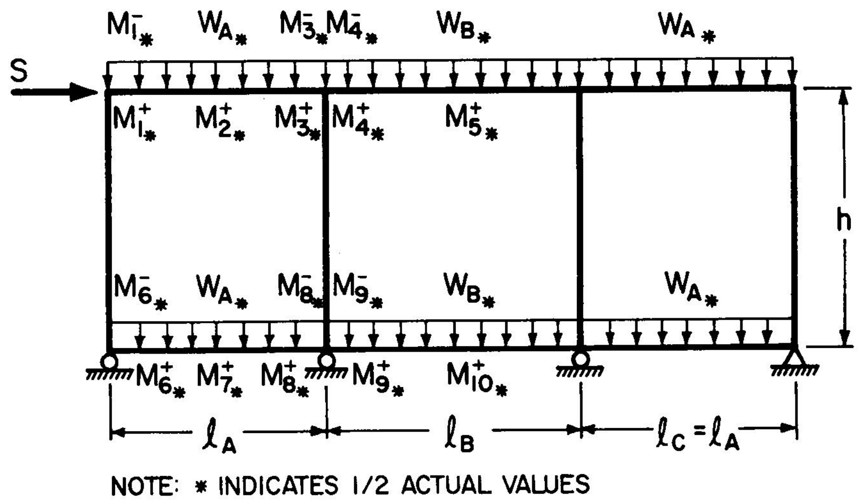


FIG. 11 FINAL DESIGN SUBASSEMBLAGE